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# Review of Topology Optimisation Refinement Processes for Sheet Metal Manufacturing in the Automotive Industry

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## Abstract

Topology optimisation is a process that is becoming increasingly reliable and necessary in the pursuit of highly efficient components comprising of low mass with a high structural performance. These components are typically mass-produced on a large-scale in automotive sectors for instance, where components are usually metallic and pressed. The ability to maximise a component's structural characteristics has yielded many variations of computational topological solvers over the years. Over time many different methodologies have been used to generate suitable manufacturable solutions. Despite this, a gap between the generation of topology optimisation solutions and the creation of ready-to-manufacture solutions still exists today. This review paper outlines existing methods for computational topology optimisation and addresses any refinement methods used to generate a manufacturable solution, particularly focussing on methodologies used in automotive sheet metal forming. These methods are scrutinised in regards to the level of manual user input needed to create a Computer Aided Design (CAD) model representation of the manufacturable solution. Suggestions are also made to highlight further work to improve these techniques for large-scale industry-standard product development.

## Keywords

Topology optimisation, Finite element analysis, Level-set method, Isogeometric analysis, Bézier curves, Post-processing, Manufacturing

## 1 Introduction

Computational topology optimisation is growing to be an increasingly reliable and necessary tool to improve quality and efficiency for development of a vast range of engineering components, for example in the manufacturing of sheet metal components in the automotive industry. Sheet metal structures for instance are typically used to create automotive components that provide significant structural support under high loads, such as bumpers, seat frames and body in white parts (SSAB 2017). Particular focus is consequently placed in the optimisation of automotive components to produce designs that both perform well and are cost-effective. The capability to create numerical models to analyse and optimise the designs of components can drastically reduce development time and allows for multiple loading scenarios to be considered simultaneously. Despite substantial advances in recent years, a significant gap remains between the creation of optimised designs in engineering analysis software and ready-to-manufacture components. This gap is usually filled with an empirical approach to correct any topological issues in the solution model such that the design complies with industry standard manufacturing processes. This approach can often be detrimental to the efficiency of the optimisation procedure and may not necessarily yield the most optimum manufacturable design. The post-processing step can be particularly time-consuming in comparison to the analysis and optimisation steps, and is highly dependent on the level of skill of the user as well as the

complexity of the model. Several commercially available (and industrially applied) optimisation solvers utilise the Variable Density Method (VDM), and are often combined with an interpolation scheme to determine load paths and optimum designs. Popular algorithms include the Rational Approximation of Material Properties (RAMP) scheme (Chen 2012), as well as the more commercially available Solid Isotropic Material with Penalisation (SIMP) interpolation scheme (Christensen and Bastien 2015), which creates solutions that redistribute the component's material densities. Consequently, the optimisation solution includes intermediate densities which cannot be manufactured, and makes the manual clean-up process more complicated. For some optimisation solvers it is therefore not possible to avoid variable densities and so the "issue" must be addressed in the post-processing stage. More recent optimisation processes include Level-Set topology optimisation (Challis 2009; Kumaravel, M. et al. 2012; Liu 2015) which creates a binary model of either full densities or no density (1s and 0s only) from variable density optimisation results. This effectively cleans up the "grey" edges of a model, creating clean lines better defining structural boundaries. Some methodologies however do not use variable density solutions, such as evolutionary optimisation, and therefore require their own methodology to refine the structure for manufacturing (different to that of a VDM solution). Alternative optimisation and refinement methods include Isogeometric Analysis using Non-Uniform Rational Basis Spline (NURBS) curves, followed by an optimisation and refinement step known as Trimmed Surface Analysis (TSA) using CAD geometry only in the solution generation (Lee et al. 2017; Kang and Sung-Kie 2016). Similar methods to TSA include the use of Bézier curves instead of B-splines, which can also be used to define cut lines but are formulated with different line properties (Lee et al. 2012). These methods are further discussed in **Sections 2 and 3**.

When considering the post-processing task of improving the manufacturability of an optimised model, it may prove difficult to create a set of parameters that work for both variable density solutions and binary solutions. This is especially true if an automated "clean-up" program is considered, in which some existing tools may work well for refining binary models but not necessarily for variable density models (and vice versa). This is evident in several examples, including Liu and Ma (2015), where only binary solutions are refined and Yi and Kim (2016), in which only VDM solutions can be refined (see **Section 3** for further detail on these methods). Despite this, it is evident from several sources, including Yi and Kim (2016) and Nana et al. (2016) that progress in the development of an automated post-processor which can correct geometry issues is being made. In spite of the promising development of these automated post-processing methods, their contribution into real-life situations and their suitability in working environments is still very limited. For instance, the majority of these programs do not produce suitable Finite Element (FE) mesh and CAD model solutions. For the limited number of unique methods that do, very few address the additional problem of generating CAD/FE solutions that represent manufacturable components, especially for sheet metal components. Recently, Zhang et al (2016) attempted to address these sheet metal considerations, but somewhat limited its progress by including its results

within the topology optimisation solver. The inclusion of component refinement within a solver can pose issues such as not being able to accommodate the different solution files mentioned previously. Integration of refinement process into the main solver may also generate results that might not be optimal, especially if post-processing refinement is considered after the solver run. From this, it is identified that a significant gap exists for component refinement that is separate from the initial solver i.e. in its own post-processor.

This review paper aims to outline existing methods for the automated clean-up of topology optimisation results and identify any contributions towards closing the gap between refined geometry and ready-for-manufacture designs. Any gaps in methodology will be identified as areas for future development. An overview of the methodologies covered includes:

- Outlining existing topology optimisation refinement methods
- Identifying any automation in the production of refined topology solution files
- Consideration of sheet metal manufacturing procedures for structural automotive components and their implementation in current topology optimisation refinement methods.

Several assumptions will be made for the identification of these methods: it is expected that any material properties used within examples will relate to linear isotropic and linear-elastic material properties due to their popular use within standard optimisation solvers. Other material properties not commonly used in the development of large-scale automotive body components, such as layered composites, will not be directly considered due to their different material characteristics (Christensen and Bastien 2015). Clean-up methods considered in this paper will solely focus on load-bearing components made from sheet metal. Some examples include B-pillars and crash boxes. Despite the common consideration of isotropic materials for optimisation processes, it must be mentioned that standard optimisation interpolation schemes such as the SIMP method can be adapted for use with non-isotropic material properties, as shown in recent works by Erik Lund, (2011 and 2009), by using SIMP methodology for composite layers. Optimisation solutions from any suitable test cases (linear static, modal, etc.) will be considered, as a focus will be made to identify methods that can refine any type of topology solution.

The remainder of this paper is divided into three sections: **Section 2** includes a **Literature Review**, highlighting the previously discussed existing methods and their implementation in topology post-processing. This will also consider suitable manufacturing methods used in the automotive industry and how these methods may be utilised within a post-processor. **Section 3** indicates a more refined overview of **Recent Implementations of Topology Optimisation & Manufacturability**, highlighting specific case studies and showcasing emerging implementations of these recent ideas. **Section 4** will draw conclusions from the processes described in **Sections 2** and **3** and indicate gaps for any **Further Research** needed to close the gap between optimisation and manufacturability.

## 2 Literature Review

This section contains an overview of leading refinement methods used for topology optimisation results. Several methodologies will be addressed in relation to their relevance towards automated post-processing. These

methods can conveniently be divided into two types of processes: ones that use a mostly mathematical basis for their methodology (**2.1**) and those which use a more heuristic (self-learning) methodology (**2.2**). Most mathematical solvers are commonly used within commercial optimisation software, such as the Variable Density Method (**2.1.1**) whereas some examples of heuristic methods such as the Isogeometric Analysis (**2.2.2**) focus on a more iterative approach, refining a solution based on the visual placement of the previous iteration. Mathematical processes generally follow a clear and systematic structure, allowing it to be implemented numerically into suitable Finite Element Analysis (FEA) software and, in some cases, even spreadsheets. This generally makes it “simpler” to prove that a given solution is indeed the global optimum, and not simply an arbitrary localised solution. This helps to explain why mathematical methods are commonly used in many commercial FEA software packages and are used to formulate optimisation solutions based on material densities or compliance. Whereas mathematical optimisation algorithms tend to find the most mathematically accurate solution, heuristic methods use a more “practical” approach. This means they follow a more experience-led methodology, in which the program “learns” how to generate optimum solutions through an iterative procedure. This allows for more practical designs to be generated but may also carry some undesirable factors such as lack of repeatability and justification (other than the “experience” of the algorithm) that a global optimum solution is generated. Several variations of standard heuristic solvers are used to alleviate these commonly faced issues. Meta-heuristic methods are a type of heuristic solver that focus on relating its solving techniques to specific “natural phenomena” such as the theory of evolution (Christensen and Bastien 2015; Christensen 2015) or more recently swarm intelligence (Kananachos et al. 2017). These methods are considered “learning-based” rather than “experience-based”, and are able to generate solutions for a greater variety of problems than general heuristic processes. Other methodologies include Neural Networks, which are designed in a similar manner to biological nervous systems and “trained” to reach an optimal solution. They tend to work by example and do not follow set methodologies as with heuristic and metaheuristic process, in which gaps are filled by making calculated assumptions (Stergiou and Siganos 2017). Despite these potential problematic areas heuristic algorithms are widely used due to their ability to produce stable structures and avoid issues such as material checkerboarding (**Section 2.1.1**). It is also possible to combine heuristic and mathematical methods (see **Section 2.1.2**) in which an iterative learning approach is used alongside fundamental mathematical methodology.

It should be noted that the mathematical and heuristic optimisation techniques in this paper relate to the usage of the techniques within a given refinement procedure. This means that the mathematical and heuristic methods identified in **Sections 2.1** and **2.2**, respectively, are all initially recognised as mathematical-based methods but in terms of when used in an optimisation solver can be further categorised into purely mathematical methods (**Section 2.1**) and heuristic, iteration-based methods (**Section 2.2**).

In order to design a truly manufacturable component, a number of different methodologies can be adopted. One option is the use of manufacturing constraints applied to the optimisation, as demonstrated via several examples in **Section 3.1**. These methods, however, mostly focus on castings rather than sheet metal manufacturing. Another option is to actively control the optimisation so that the developed topology can only utilise certain types

of geometry. (Zhang, Norato et al 2016) is an example of this type of approach. Regardless of which method is used, it may be beneficial to utilise optimisation-specific data, such as element sensitivity numbers, as opposed to only considering the final geometrical result. This may provide a better datum point for any refinement methodology and can help identify methods useful for improving designs that utilise specific manufacturing methods including sheet metal forming.

Both mathematical and heuristic variations will be critiqued for their ability to create refined solutions that closely represent a manufacturable model. The focus will then be directed towards automotive industry manufacturing methods, specifically sheet metal forming, identifying the methodologies used to create a model that represents a manufacturable solution.

## 2.1. Mathematical Optimisation Approaches

The following subsections identify existing optimisation and refinement methodologies that show significant inclusion of mathematical operators and matrices used within the basis of its solver operations. The methods identified within this section are most commonly seen in commercial optimisation solvers, to which variants of the formulae mentioned are currently being included in new emerging optimisation and refinement codes.

### 2.1.1 Variable Density Method (VDM) and Solid-Isotropic Material with Penalisation (SIMP)

The Variable Density Method (VDM) is arguably the most widespread topology optimisation solver due to its use in many commercial FE software packages. This method alters each individual element's density in order of its structural importance. In order to derive an optimisation solution, it is necessary to interpret structural response data using FEA. The FEA process considers three key variables: the applied load,  $F$ , the displacement of each node,  $U$ , and the material stiffness,  $K$ , which can be represented in matrix form:

$$\{F\} = [K]\{U\} \quad (2.1)$$

In which the stiffness matrix,  $[K]$ , is defined by using relevant material data from the structure, i.e. Young's Modulus and Poisson's ratio and calculated locally for each element, allowing for a representation of the load distribution to be made. The applied structural load will allow for the user to "visualise" the distribution of forces along a system, which can in turn be viewed in existing post-processing software. By inverting the matrix and solving for the material stiffness,  $[K]$  (assuming all values for  $\{F\}$  and  $\{U\}$  are known), the identification of which individual elements are most influential to the structural performance, or dynamic response, can be determined. This information can then be used in the optimisation process in which element densities will be redistributed. Redistribution can be calculated through computational methods, with the most common of methodologies being topology optimisation. This process typically removes parts of a structure that are not as "efficient" in resisting the external load as other elements, and aims to improve component design without compromising on structural performance. Structurally important elements are usually determined by their compliance (inverse stiffness) in which it is preferred that the stiffest structure is created from the least material specified by the user. Topology optimisation is firstly defined by identifying an objective function, that is, a function that provides the overarching aim of the re-design. This objective function generally takes consideration of either the Young's Modulus,  $E$ , or the mass density,  $\rho$ . Once the objective function is decided from these two options, the user will then identify a single (or multiple) design variables. These variables

are measurable parameters which are controlled by the solver in order to achieve the target stated in the objective function. Due to the ease of which it can be "visualised" by the user, it is very common within various software to consider mass density as a design variable, by which it is commonly reduced in order to lightweight the structure (and consequently lower the number of elements). The initial problem attempts to generate a solution in a binary format, either eliminating an element (represented as zero density) or to keep the element (represented as full, i.e. 1, material density). **Equation 2.2** demonstrates the process taken, by which the compliance tensor, " $C_{ijkl}$ ", is reduced (stiffness is increased):

$$C_{ijkl}(y) = \rho(y)C_{ijkl}^0 \quad (2.2)$$

Where:  $\rho(y) \in [0, 1]$

$\rho(y)$  refers to the density of an individual element,  $y$ , which is commonly used as a design variable for optimisation processes. Furthermore, the subscript,  $ijkl$ , represents the individual stress tensor ( $i, j$ ) and strain tensor ( $k, l$ ), in matrix notation, and  $C_{ijkl}^0$  denotes the "original" and constant material properties. The constitutive tensor is generally identified as a function that equates component stresses and strains. The most general representation of a constitutive tensor can be that for isotropic linear materials and can be represented in matrix form. The basic representation of this tensor and its relation to material stresses and strains for isotropic material behaviour is shown in **Equation 2.3**:

$$[\varepsilon] = [C][\sigma] \quad (2.3)$$

From **Equation 2.3**,  $[\varepsilon]$  represents the strain tensor and  $[\sigma]$  is the stress tensor of the isotropic material. These matrices can be expanded to show the variables needed for their derivation as follows (Wikiversity 2017):

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix} \quad (2.4)$$

From **Equation 2.4**,  $E$  denotes the Young's Modulus of the material,  $\nu$  is the material's Poisson's ratio and  $G$  is the shear modulus of the material, which can be calculated by using the relationship as follows:

$$G = \frac{E}{2(1+\nu)} \quad (2.5)$$

This formulation (for binary solutions) is to be performed for each element individually. Additional iteration steps may also be needed such that this process is repeated, however this is subject to the requirements set by the user. **Equation 2.2** shows that  $\rho(y)$  for each element can take the value of either 0 or 1, indicating that the element in question is either kept (1) within the solution or removed (0). Assuming linear elastic material properties, the two independent variables representing the material properties that define the compliance tensor,  $C_{ijkl}(y)$ , are  $E$  and  $\nu$ , as previously stated. As  $E$  is the more influential of the two parameters many solvers consider  $\nu$  to be constant, thereby reducing the number of design variables to one (per element). In relation to element strain energy, **Equation 2.6** can be used to represent the optimisation problem as a function of density,  $f(\rho)$ . Solving this will generate a number larger than 0 but less than or equal to 1:

$$f(\rho) = U(\rho) = \frac{1}{2} \int (\{\varepsilon\}^T [E(\rho)] \{\varepsilon\}) dV \quad (2.6)$$

The function is integrated along the entire volume of the component (i.e. all elements in the model).  $U(\rho)$  denotes the linear strain energy (compliance), and  $\varepsilon$  represents the strain acting on the component.

After identifying the formulation for the material density generation, it is necessary to put this into context of the previously mentioned objective function for the optimisation problem, as well as defining any suitable design variables placed on the problem. The general definition of structural optimisation is often described as a structure that performs at its best and most efficient (Christensen and Bastien 2015). So it can be assumed that any design is to have its weight minimised to the best of its ability whilst at the same time retaining its best structural performance. Therefore, on the most basic of understandings, the objective function for a general structural optimisation process can be expressed as:

$$\min(f(x)) \quad (2.7)$$

Where  $f(x)$  represents any function, or more specifically, the problem to be optimised. The function  $f(x)$  could be equal to any value and would usually represent the volume or mass of the system. Solutions can converge to a variety of suitable answers, though it can be assumed that a zero volume solution is not sufficient when performing a structural optimisation. This is simply because a zero solution is practically non-existent, thus its initial structural function is not achieved. To alleviate the issue of non-feasible solutions, it is necessary to introduce suitable design variables that can limit the number of generated solutions. This can therefore change the optimisation problem from an “open-ended” or “multimodel optimisation” solution to a more discrete solution. For example, if a non-zero solution is required and any integer below 0 is not feasible, a design constraint can be added to the end of **Equation 2.7** to create **Equation 2.8**:

$$\min(f(x)) | x > 0 \quad (2.8)$$

Additionally, the optimisation problem stated in **Equation 2.8** can be further constrained by including more design variables. These can be applied indirectly (relating to another external variable independent of  $x$  such as stresses or displacements) or directly (as identified by constraining  $x$  as shown in **Equation 2.8**).

As indicated in **Equation 2.6**, any generated solution using this topological optimisation method will produce a mesh model that consists of full density elements and gaps where lower/negligible density elements were removed. This is true if the rounding methodology shown in **Equation 2.2** is applied. It is however important to distinguish the change of individual element densities (highlighted in **Equation 2.2**) to the calculation of the density for the overall system (**Equation 2.6**). For the overall system, the optimal density is defined in relation to global strains and loads. Whereas this may be an easy-to-implement solution, e.g. when maximising the structural stiffness, several issues may still arise. The most common issue is that certain elements may be removed in a manner such that the design is not easy to manufacture, consisting of holes or disconnected sections that would not be easy to replicate in real-life: this is known as the checkerboard effect. In the worst case scenario the optimised structure consists of elements that are still connected by their nodes but not attached to elements on their immediate sides, as depicted in **Figure 1**:



**Fig. 1** – An example of material checkerboarding. The left and right sections are not suitable for manufacture as the elements are not connected by adjacent elements (adapted from Designer.mech.yzu.edu.tw 2017)

In order to reduce the checkerboard effect the allowable values of the individual element density  $\rho(y)$  as defined in **Equation 2.2** could be relaxed. Allowing intermediate values between 0 and 1 would enable the creation of continuous structures, reducing the checkerboard effect. This effectively takes the example shown in **Figure 1** – known as a “Binary Solution” – and creates a “Variable Density Solution”. This can be achieved through the use of the SIMP interpolation scheme. This method allows intermediate densities, but attempts to “guide” each element towards the desired 0-1 density by penalising intermediate density values. This creates a model with elements that are not all in a binary format, but instead generates elements with intermediate densities that are relatively close to the 0-1 representation. This level of relaxation can be manually altered and is dependent on user preference and experience. In this case, **Equation 2.2**, in its relaxed form, will now become **Equation 2.9**

$$C_{ijkl}(y) = \rho(y)^p C_{ijkl}^0 \quad (2.9)$$

$$\text{Where: } p \geq 1 \wedge \rho(y) \in [0: 1]$$

Typically, SIMP interpolation schemes are used to minimise mass or minimise compliance (maximise stiffness). The latter can be defined by considering the following formulation:

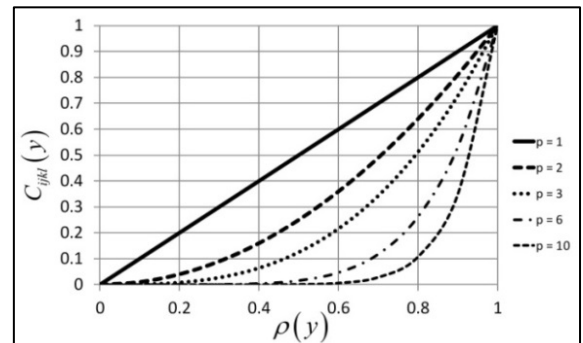
$$\min c(\rho) = U^T K U = \sum_{i=1}^n E(\rho_i) u_i^T k_0 u_i \quad (2.10)$$

From which  $k$  and  $u$  represent local element stiffness and displacement matrices, and the notation  $T$  refers to the transpose of those matrices (Sigmund 2001). This formulation is subject to several criteria, such as that the material density must be greater than zero and less than 1 of the initial density. This is further explained through the inclusion of an “optimality criteria”, in which material densities are defined by conforming to the following:

$$\rho_i^{\text{new}} = \begin{cases} \max(0, \rho_i - m) & \text{if } \rho_i B_i^\eta \leq \max(0, \rho_i - m) \\ \min(0, \rho_i + m) & \text{if } \rho_i B_i^\eta \leq \min(1, \rho_i + m) \\ \rho_i B_i^\eta & \text{otherwise} \end{cases} \quad (2.11)$$

In which  $m$  is a moving limit set for the change in density,  $\eta$  is a damping coefficient ( $=1/2$ ) and  $B$  is an optimality condition that is separately formulated.

The penalisation factor  $p$  in **Equation 2.9** can be adjusted for each optimisation task. An example of how  $p$  influences element stiffness on a scale of 0-1 is illustrated in **Figure 2**.



**Fig. 2** - Influence of Penalisation Factor on Density Relaxation (Christensen and Bastien 2015)

As shown, by increasing the penalisation factor,  $p$ , the more likely the density solution will “force” the solution towards either 0 or 1, thus limiting (but not necessarily preventing) densities close to 0.5 from being generated. By using a penalisation factor of 1 however, the solution is not relaxed and will not be directed towards a specific extreme. It should be of interest that, as indicated in **Figure 2**, when  $p$  increases, the slope defined will become much steeper. This could potentially lead to model instabilities due to that any slight variation in material density,  $\rho(y)$ , can drastically change the computed stiffness (Christensen and Bastien 2015).

As previously defined, it is assumed that the general relationship between stress and strain for linear elastic and isotropic materials is represented by Hooke’s law; i.e. the deformation of an element is directly proportional to the load applied to it. Based on **Equation 2.6** it is also assumed that the Young’s Modulus and strain of the material are the only variables to be considered in the initial topology solution generation. Despite this linear relationship of stress and strain, several external factors such as component stiffness are not explicitly accounted for. Artificial (geometrical) stiffness may be an issue which can occur when an element with negligible (material) stiffness is still present in the model (Christensen and Bastien 2015). As the VDM SIMP methodology does not delete elements, the otherwise present negligible density element will still impose its own constraints on the component, thus generating additional structural stiffness on neighbouring elements. This issue is not accounted for in the otherwise linear Hooke’s Law relationship and may show differing material stress recordings from the actual real-life values. This is particularly true when additional refinement stages are involved that require referring back to stress values, such as in optimisation processes that require multiple iterations of the VDM solver (see **Section 3.3**) (Kang and Youn 2016). As most of the stiffness influence in the element is controlled by its material properties, the induced stiffness due to geometry constraints may only be small in comparison. Nonetheless it should be aware to the user that these discrepancies do exist when using this methodology (Christensen and Bastien 2015).

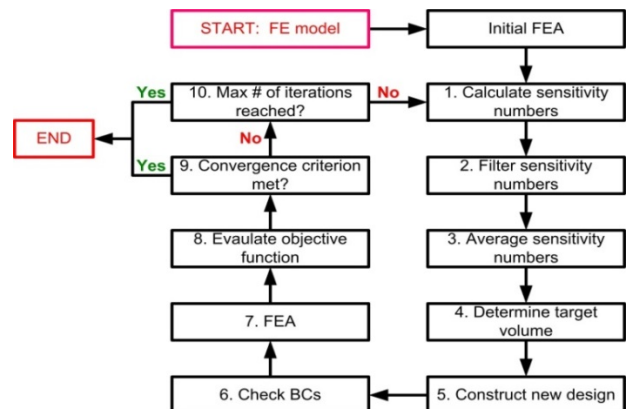
## 2.1.2 Evolutionary Optimisation Methods

Evolutionary optimisation methods are a group of optimisation procedures that draw inspiration from nature-inspired methods such as the theory of evolution. These methods are often coupled with FEA processes, such as those mentioned in **Section 2.1.1**, and follow a more heuristic approach to generating a solution (that being a more “practical”, learning-led approach as opposed to a purely analytical method). This combination of learning-led and analytical approaches can be described as a meta-heuristic methodology, with the fundamental FEA process being used alongside nature-inspired methods (Christensen and Bastien 2015). Key differences from this and the standard SIMP methodology is that evolutionary optimisation does not consider the use of variable density element solutions, meaning that a binary solution file is created with no intermediate densities. A solution is obtained by removing a controlled quantity of elements from the FEA model per iteration from the design domain, removing elements deemed insufficient from the model in stages (Tanskanen 2002). Several variations of Evolutionary Structural Optimisation (ESO) methods exist, but for the purposes of this document the more commonly used and studied methods, ESO, Additive ESO (AESO) and Bi-directional ESO (BESO) will be discussed in this paper. These methods may also be seen as a type of heuristic method, with some processes being described as meta-heuristic as they are based on the theory of evolution; removing or

adding elements based on the principle “survival of the fittest”. It should be noted that most evolutionary methods identify a specific threshold at which elements are removed during the iterative process. This is to reduce the likelihood of instabilities occurring if too many elements are removed in a single iteration. These and are generally small ratios such as  $1/10^6$  of the initial structural volume (Tanskanen 2002). The underlying methodology in these solvers, however, still relies on a mostly mathematical procedure in the form of an FE analysis step, defining structural performance and removing elements deemed “unnecessary” for its load case.

The general ESO methodology revolves around a similar process to that of the Variable Density Method, in which optimisation results are informed by FEA results of the structure. A key difference would be that the FEA step is performed for every iteration. The process firstly consists of the declaration of an FE model, followed by the definition of a rejection ratio,  $RR$ , and a performance index,  $PI$ . A rejection ratio is a defined criterion for how many insufficient elements will be removed per iteration, and can be dependent on several criteria such as element stresses or strains. As mentioned in **Section 2.1.1**, there will be no artificial stiffness generated in any solution files as ESO only uses binary solutions. Issues may still arise as binary models may encounter component checkerboarding as previously discussed (**Figure 1**). Issues that can arise from the use of ESO include the uncertainty of model stability once an element is deleted. As the model is slowly optimised during the iterative process, there is no method of retrieving lost elements once they have been removed. This could cause some solutions to converge to unusable designs. This issue has however been addressed through several methods, one of which being BESO.

An alternative methodology to that of ESO is known as Additive Evolutionary Structural Optimisation (AESO), which follows the same principle to that of standard ESO but instead starts with an empty design space and adds elements into that space (as opposed to starting with a full model and reducing it). Both ESO and AESO follow similar procedures and also encounter similar issues when forming a structural solution. BESO attempts to alleviate the main limitation of ESO and AESO, where the process is limited as to what can be changed during iterations, due to them working with “non-complete” structures. This includes making three main considerations to correct this issue, including a filtering scheme, an improved sensitivity analysis and, from its namesake, a new method that involves either removing or adding elements throughout the iterative process. BESO utilises the methodologies from both ESO and AESO, by both adding and removing elements over the iterative process (Xie 1998). **Figure 3** illustrates the main BESO algorithm, with the inclusion of these new steps:



**Fig. 3** – Bidirectional Evolutionary Structural Optimisation (BESO) Algorithm (Christensen and Bastien 2015)

The improved filtering scheme and sensitivity analysis aim to determine the influence the individual FE elements have on the main objective function. In relation to an individual element,  $x$ , the new objective function for the sensitivity analysis can be defined as shown in **Equation 2.12**:

$$\Delta f(x) = \lim_{n \rightarrow \infty} \sum_{i=1}^n (E_i^e - E_{i-1}^e) = -E_n^e \quad (2.12)$$

From **Equation 2.12**,  $E$  represents the strain energy and  $e$  identifies that this strain energy is in relation to a specific, single element.  $i$  denotes the interval placed on the design displacement and  $n$  identifies the total number of intervals for the design displacement. From this, it can be stated that  $E_n^e$  is the total strain energy of the removed element,  $y$  - as denoted in **Equation 2.2**). It can also be noted from this equation that the decrease of total external work from the removal of one element is equal to the total strain energy of the element in its final deformed state and is not influenced by the size of displacement intervals (Huang and Xie 2008). Furthermore, Huang and Xie (2008) also defined the sensitivity of the individual element within the structure as:

$$\alpha_i = E_n^i \quad (2.13)$$

The process by which element sensitivities are considered within the BESO formulation is known as the "hill-climb" method or the "steepest descent" algorithm. This allows elements to be removed or added in accordance to their calculated sensitivity value, with the higher numbers causing elements to be added and lower values correlating to their removal (Huang and Xie 2008).

The second updated parameter for BESO is that of a new filtering scheme. This process can be defined by firstly identifying a sensitivity number for the optimised component as follows:

$$\alpha_i = \frac{\sum_{j=1}^N \omega(r_{ij}) \alpha_j}{\sum_{j=1}^N \omega(r_{ij})} \quad (2.14)$$

From this,  $N$  can be described as the total number of elements in the entire mesh, and  $\omega(r_{ij})$  represents a weighting function defined through **Equation 2.15**:

$$\Omega(r_{ij}) = \frac{\exp\left(-\left(\frac{(r_{ij})^2}{2\left(\frac{r}{3}\right)^3}\right)\right)}{2\pi\left(\frac{r}{3}\right)} \left| \{i \in N | r_{ij} \leq r\} \right| \quad j = 1, 2 \dots N \quad (2.15)$$

From **Equation 2.15**,  $r_{mn}$  represents the distance between the centres of two adjacent elements labelled  $m$  and  $n$  and indicates that there is a degree of neighbourhood influence when adding or removing individual elements. This procedure acts as a sensitivity filter which can serve two main purposes. Firstly, the formula will allow for the generation of a global sensitivity value, allowing for this data to be extracted from the design domain. Secondly, the consideration of neighbouring element locations will be correlated to any component checkerboarding and attempt to manipulate values in order to avoid this (Huang and Xie 2008).

Whereas BESO promises better result quality than that of ESO/AESO formulation, several other issues are also present. These predominantly consist of the increase in computational cost due to the algorithm needing to perform an FE analysis for every iteration step. Although this is also the case for VDM-SIMP based optimisation, the significant difference is that the stiffness matrix needs to be recreated for ESO/BESO because elements have been removed from and/or added to the structure. Assuming implicit FEA this also means that the new

stiffness matrix needs to be re-inverted for each iteration of the optimisation, which is generally a very CPU and memory intensive task (Christensen and Bastien 2015) (Huang and Xie 2008). Additionally, despite being generalised as a mostly mathematical procedure, both ESO and BESO can be classed as heuristic solvers. This in turn can reduce the accuracy of the optimised solution, and is deliberated by some sources that no detailed proof of accuracy is established in the "optimised" (Rozvany 2007).

## 2.2. Heuristic Optimisation Approaches

The following subsections identify existing optimisation methodologies that, unlike the previously mentioned methods, are more commonly seen used in heuristic-based optimisation or model refinement solvers. It should be noted that all methodologies identified in **Sections 2.1** and **2.2** hold significant mathematical basis, though it can be argued that the methods identified in this section show greater usage in heuristic-based solvers. This can be identified through recent papers such as an example by Kang and Youn (2016), to which isogeometric analysis techniques are updated using an iterative process to achieve a more refined structure. Further details of this paper's methodologies are explained in **Section 3.3**. Therefore, it can be assumed that whereas no direct mentions of iterative loops or specific common heuristic methodologies may be mentioned within these subsections, the mathematical bases described will highlight methods commonly used in heuristic solvers and refinement post-processing.

### 2.2.1 Level Set Method for Topology Optimisation

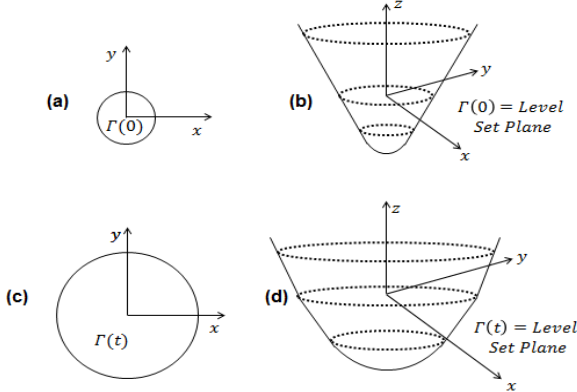
The level set method (LSM) is an image recognition technique that is used to track geometries using grids and shapes as reference points. It has also been adapted for the refinement of optimisation results, in which holes and shape features can be identified within a topology optimisation program. The process includes the creation of a plane (original front) that cuts across a section of a component, with geometrical features identified for the cut section. The method was originally developed by Stanley Osher and James Sethian (1988) as an image recognition technique for geometric components but has more recently been integrated into topology optimisation solvers, such as by Challis (2009). Level set methods are designed to solve models that are geometrically mapped through several methods, being either traditional FEA density meshes, conforming meshes (only the material domain being discretised) or boundary based grids (a uniform grid with local boundaries enforced around edges) (van Dijk et al. 2012). The level set method's ability to identify geometric features of components has been used to define edges of variable density FEA models, removing any greyscale elements around holes and borders. These borders are defined using a level set function,  $\phi$ , with the following properties:

$$\begin{aligned} \phi(x, t) &> 0 \text{ for } x \in \Omega \\ \phi(x, t) &< 0 \text{ for } x \notin \Omega \\ \phi(x, t) &= 0 \text{ for } x \in \partial\Omega = \Gamma(t) \end{aligned} \quad (2.16)$$

**Equation 2.16** identifies the open region,  $\Omega$ , which is defined as the area outside of the physical component space, i.e. outside the boundary layer of the component.  $x$  and  $t$  refer to the position and time, respectively, of the open region,  $\Omega$ , when it is subject to a velocity field,  $v$ . This means that the values for  $x$  and  $t$  will change when  $\Omega$  moves up or down the contours of the object in question, in which the 2D boundary layer will change due to its different position along the object (Osher and Fedkiw 2001). The level set function defines a border, with locations inside or outside of the border being



represented as positive or negative values, respectively. A border is defined as a set of pixels (elements) that are represented by the value 0. This is also known as the zero level set of the curve,  $\Gamma$ , which will be updated with each iteration of the solver (Osher and Fedkiw 2001). The level set equation is represented in **Equation 2.17**, in which it states that the boundaries are re-defined along with the velocity field,  $v$ . This velocity field is used to describe a variety of functions for the normal motion of the level set plane,  $\phi$  along the surface, including its curvature and normal direction. Whereas **Equation 2.16** identifies the requirements of the boundaries in a static state, it is desired that this function is manipulated to consider the evolution of this plane when it moves along the normal of the surface over time (see **Figure 4**). Knowing from **Equation 2.16** that  $\phi=0$  for all situations on the boundary surface, this relation (**Equation 2.17**) can then be integrated using the chain rule (**Equation 2.18**):



**Fig. 4** – Propagating circle (bounding layer) moving normal along a cone: (a) and (b) illustrate the level set plane and cone, respectively; (c) and (d) illustrate a larger bounding layer for motion,  $v$  over time,  $t$  (adapted from Sethian (1994))

$$\phi(x(t), t) = 0 \quad (2.17)$$

When integrating **Equation 2.17** using the chain rule, the statement becomes:

$$\phi_t + \nabla\phi(x(t), t)x'(t) = 0 \quad (2.18)$$

As previously mentioned, the normal velocity of the moving plane is represented by  $v$ . Knowing that the change in distance over time is represented in **Equation 2.18** as  $x'(t)$ , derivation of the updated level set evolution formulation can be attained as shown in **Equation 2.19**:

$$\frac{\delta\phi}{\delta t} = v|\nabla\phi| \quad (2.19)$$

From **Equation 2.19**,  $\frac{\delta\phi}{\delta t}$  identifies the change of the level set plane over a set time,  $v$  is the velocity of the level set plane in the normal direction and  $|\nabla\phi|$  represents the modulus of change in position of the level set plane (Sethian 1994).

Recent developments of the level set method can also be implemented into topology solvers, such as a process known as the Relaxed Level Set which allows for the creation of holes instead of only identifying them (Sigmund and Maute 2013). The level set method has more recently been implemented into topology optimisation solvers, acting as an additional mesh refinement step at the end of each iteration process. In most instances, the solver follows a main iteration loop pertaining to the topology optimisation, and is followed by a design update stage relating to the level set methodology. An example of this implementation can be found in the works of Challis, V. J. (2009), in which a 129 line code is generated for a level set topology optimisation solver. This code is based on the 99-line

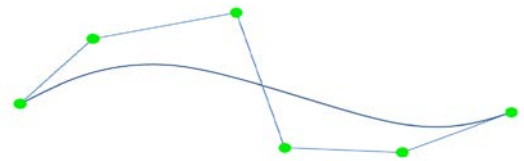
Matlab code by Sigmund, with several modifications to include the level set refinement step (Sigmund 2001).

Even though level set topology optimisation is proven to refine optimisation results by generating more discernible component boundaries, its implementation into key automotive component generation seems relatively ambiguous. For instance, the refinement process may create a binary model but does not consider manufacturing methods such that an optimised solution can be derived with no additional manual corrections. This indicates that level set topology optimisation is an important tool for removing manual user input in component post-processing but does not include all requirements to make the system fully autonomous.

## 2.2.2 Isogeometric Analysis (IGA) with Trimmed Surface Analysis (TSA)

Isogeometric Analysis (IGA) is a newly developed method used as an alternative to conventional FE procedures. It attempts to create an analytical solution without the need to create a mesh as with “conventional” FE solvers. Instead, NURBS curves are used to represent the model in the analysis process.

A NURBS curve (Non-Uniform Rational Basis-Spline) is a commonly used computational method to mathematically represent a curved line or surface in an environment using computer graphics software. These curved lines are commonly referred to as splines, and are mathematically formulated with definitions of certain geometrical characteristics. Splines are generally categorised into degrees of 1, 2, 3 or 5, which can represent lines, circles, and higher degrees of free-form lines, respectively. NURBS curves may also be referred to by their order number, which is the value of the degree of the curve plus one (rhino3d 2017). NURBS curves consist of line components that are controlled using a series of pointers, known as control points, which are positioned either side of the curve. These control points “pull” the curve towards their location, thus creating a curved line representation. The user can also manually influence desired sections of the curve by simply moving any of the control points. This highlights the fact that the control points work on a local coordinate basis, allowing the user to change single points without greatly influencing a change to the entire curve. An example of the generation of a NURBS curve is shown in **Figure 5**.



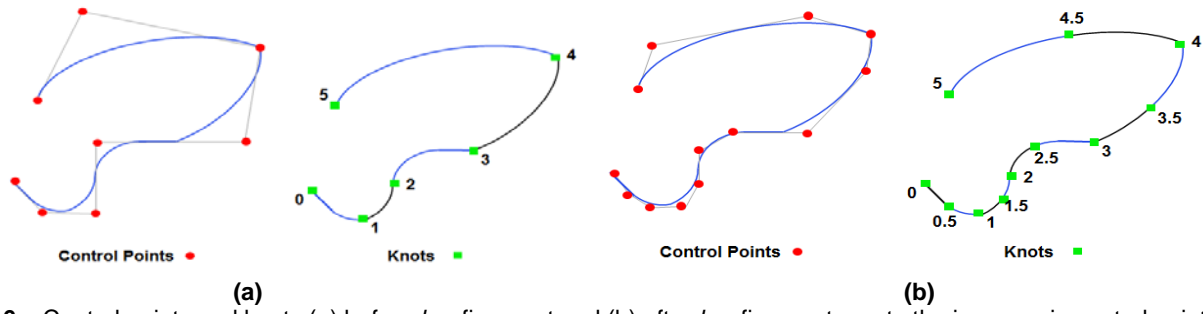
**Fig. 5** – Generation of a NURBS curve line, with points representing “control points” (adapted from wikipedia.org 2017)

Formulation of the creation of a NURBS curve in 3D geometrical space and  $n$  number of control points is defined in **Equation 2.20**. Here, it is assumed that the control points and corresponding weighting values increase by order of 1 in integers up to the  $n^{\text{th}}$  value ( $P_1, P_2 \dots P_n$  for control points and  $w_1, w_2 \dots w_n$  for corresponding weights).

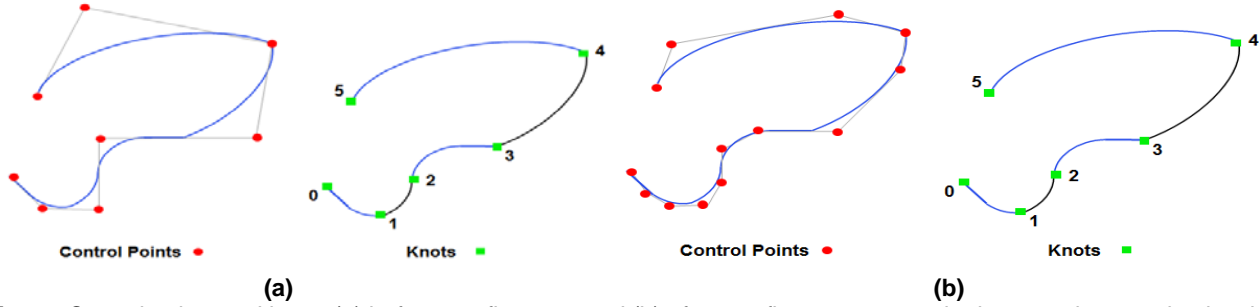
$$C(t) = \frac{\sum_{i=1}^n N_i(t)w_i P_i}{\sum_{i=1}^n N_i(t)w_i} \quad (2.20)$$

From **Equation 2.20**,  $N_i(t)$  is the b-spline function for the  $i^{\text{th}}$  iteration. The degree of the curve is  $m$ , and consequently its order is  $k=m+1$ . The number of knots in the structure is defined as  $n+k: t_1 \dots t_{n+k}$ . If all of the corresponding weighting factors are equal, they can be cancelled out of this equation, with the mathematical





**Fig. 6** – Control points and knots (a) before  $h$ -refinement and (b) after  $h$ -refinement – note the increase in control points and knots (adapted from Lovadina et al. (2017))



**Fig. 7** - Control points and knots (a) before  $p$ -refinement and (b) after  $p$ -refinement – note the increase in control points but same number of knots (mesh size) (adapted from Lovadina et al. (2017))

representation of the NURBS curve simplifying to that in **Equation 2.21** (Math.stackexchange.com 2017):

$$C(t) = \sum_{i=1}^n N_i(t)P_i \quad (2.21)$$

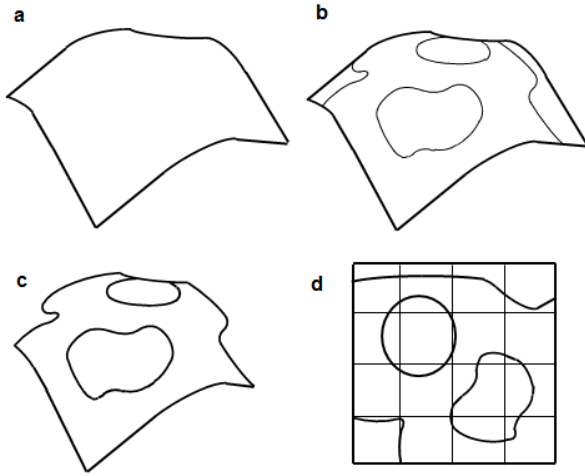
After NURBS curves have been used to generate clearly defined geometrical lines, they can be used to generate NURBS surfaces, which in turn can be used to define 2 and 3-dimensional components. These NURBS generated components will then be used to accurately represent a component or system as a CAD model. This model can subsequently be used for IGA to analyse the structure. Unlike conventional FE methodology, the CAD model will be used for the structural analysis instead of generating a mesh to replace the CAD model. A mesh will however still be used for the analysis in which a grid will be “projected” over the existing CAD, with no distortions made to the original geometry. After defining an initial mesh to grid the CAD surfaces, a set of three refinement methods can be used on the mesh to improve the accuracy of the later-to-be-performed analysis. The methods are known as the  $h$ ,  $p$  and  $k$ -methods and involve adjusting several aspects of the NURBS geometry, such as the number of control points or the number of knots (points that define where a line connects with another). These mesh refinement methods aim to improve the result approximation by more accurately representing geometry.  $h$ -refinement focusses on altering the “mesh” of the geometrical NURBS spline. This is achieved by increasing the number of knots (and therefore control points) of each individual curve. This in turn allows for greater geometry approximation by providing a greater level of freedom to edit and adjust knot positions, making the contours more clearly defined.  $p$ -refinement uses a different technique than that of  $h$ -refinement, in that it does not change the number of knots on the NURBS curve, leaving the design with the same mesh size. Adjustment is then focussed on enlarging the approximation space by increasing the number of control points on each individual node. This provides the design with a greater ability to adjust the geometrical position of the knots but does not increase computational cost as much as  $h$ -refinement, due to no additional knots being created (therefore no increase in mesh size). An appreciation of  $h$  and  $p$  refinement in visual terms can be seen in **Figures 6** and **7**.

$k$ -refinement is another methodology that can improve the quality of the geometrical NURBS structure. It utilises a relationship between the number of knots in a curve and the order of the curve, and ensures that this ratio is kept when either variable is altered. It can be assumed for all NURBS curves that when there is a curve of order  $p$ , the derivatives that can be defined from this function are of the order  $p-1$ . When the order of the curve is increased, this ratio is preserved for all situations. For instance, if the order is increased by a factor of  $q$  and a unique knot value is provided, the number of possible derivatives will always be treated as  $q-1$  of the order number (Hughes et al. 2005). This methodology can constitute to the derivation of higher order partial differential equations that can be used to create smoother curve representations. This leads to the definition of collocation methods that can be performed when only utilising the IGA process (Auricchio et al. 2010, 2012).

By generating a usable refined CAD surface, the main IGA process is then performed. This process runs in a very similar manner to the VDM procedure (**Section 2.1.1**), in that it runs a finite element analysis on the structure to determine stresses, strains, modal frequencies and more. The key difference between these two methods is that the IGA uses the CAD structure with a uniform parametric grid instead of the standard FEA mesh used in VDM.

After generating a computational analysis using IGA, an optimisation of the geometry can be performed. This may for example be achieved using a methodology known as Trimmed Surface Analysis (TSA). The method initially proposed by Kang and Youn (2016) promotes the idea of removing parts of a surface over each iteration, updating and removing material based on where the least stressed areas of the model are. This is achieved by using the data from the IGA stress analysis, in which a NURBS surface is generated using the formulae showcased in **Equations 2.20** and **2.21**. This surface is created using the stress data from the IGA and creates a “cut” line/surface that lies over the original CAD. The new cut surface is positioned where the least stressed sections are, prompting for these areas to be “cut” and removed. These two surfaces are then read together and from this both a physical domain (CAD model representation) and

a parametric domain (flat 2D surface of cut shape with no curvatures to the surface) are generated. Using only the flat parametric surface, a grid is placed over the flat geometry in the same manner to that of the previously outlined IGA (Kang and Youn 2016). Visual representation of this process is shown in **Figure 8**:



**Fig. 8** – a) IGA NURBS surface b) TSA cut lines added c) New cut surface and d) parametric grid representation of new surface (adapted from Kang and Youn (2016))

Initial consideration of a method that does not utilise a mesh in a conventional FE sense raises concerns relating to the overall accuracy of the method. Conventional meshing procedures will require the generation of elements with minimal distortion to prevent instabilities in relation to the stiffness matrix derivation. Additionally, based on the limited amount of published material in this field of research it is not known whether or not obtaining the analysis results from IGA are computationally less expensive than those obtained using conventional FEA. The ability for this method to generate a solution for specific manufacturing processes is also ambiguous, as there is currently no discernible link between this new methodology and its ability to improve designs in, for instance, the automotive industry. Instead, it can be shown that IGA highlights a relationship between CAD solution creation and its link to FEA solution files. Particularly, it identifies the manipulation of CAD geometry such that it can perform in a similar manner (potentially with better accuracy) to that of standard finite element mesh results.

## 2.2.3 Bézier Curves

Bézier curves are another variation of geometric curve generation that can be used to create CAD models suitable for topology optimisation. They can be considered as a counterpart to NURBS curves in that they both have the same general concept of computational line and subsequent surface generation. The primary difference between the two types of curves is found within the methodology used to create them. Bézier curves consider the use of linear interpolation to define a curve or surface, meaning that several “lines” are generated along two crossing axis which collectively represent a curve (see **Figure 9** for clarification). It can therefore be established that the curve lies inside the original set of control points and do not act in the same pulling manner as with NURBS curves and surfaces (courseware.deadcodersociety.org 2017).

Bézier curves are generally identified by their order (degree) number,  $n$ , which is defined as a curve that has  $n+1$  control points in which to define it, the generation of

a Bézier curve can be formulated as displayed in **Equation 2.22**:

$$P(u) = \sum_{i=0}^n P_i B_{i,n}(u) \quad 0 \leq u \leq 1 \quad (2.22)$$

$P(u)$  refers to any point on the curve, with  $P_i$  representing a control point.  $B_i$  is known as the Bernstein polynomial and is defined as the basis, or blending, function for the curve as follows:

$$B_{i,n}(u) = C(n,i)u^i(1-u)^{n-i} \quad (2.23)$$

Where  $C(n,i)$  is the binomial coefficient:

$$C(n,i) = \frac{n!}{i!(n-i)!} \quad (2.24)$$

In which  $i$  is an integer outcome from an  $n$  degree order curve. By using the method of binomial expansion, the equation is then defined as an expanded equation as shown:

$$\begin{aligned} P(u) = & P_0(1-u)^n + P_1C(n,1)u(1-u)^{n-1} \\ & + P_2C(n,2)u(1-u)^{n-2} + \dots \\ & \dots + P_{n-1}C(n,n-1)u^{n-1}(1-u) + P_nu^n \end{aligned} \quad (2.25)$$

where  $0 \leq u \leq 1$

By the definition of the expanded **Equation 2.25**, it is understood that the linear interpolation occurs about the two end control points. This can be further proven by substituting the limits for  $u$  (0 and 1) into the equation. It is important to note that only the first and last control points lie on the curve itself: the other control points “guide” the shape, derivatives and order of the curve. Additionally, the curve is always tangent to the first and last control points. Because of this, the curve shape tends to “follow” this pattern (see **Figure 9**). Control points can be manually adjusted by applying a multiplying factor to the control point in question. This value can be adjusted by the user manually; when the curve is pulled closer towards the updated control point, the greater the multiplying factor is (Zeid and Sivasubramanian 2010).

Surfaces can also be generated in a different manner than that of NURBS surfaces, with an enclosed Bézier curve forming what is known as a “bounding box”, which can then be integrated into a suitable programming “loop” creating the surface (Deadcodersociety.org 2017).

Despite Bézier curves being widely used within computer graphics and modelling for several years, their implementation in topology optimisation methods has only very recently been considered. Examples of recently proposed methodologies work in a similar manner to those indicated in the previously mentioned Isogeometric Analysis (**Section 2.2.2**), where the CAD model takes priority to any FE mesh used. A paper by (Lee et al. 2012) proposes a methodology involving simultaneously updating the CAD geometry and FE mesh, with topological changes applied to the CAD model first and foremost. Lee et al. (2012) also follows with a case study which utilises the method for optimising the topology of an automotive metal casting die. The process of updating the CAD and later the FEA mesh is continued throughout each iteration process until convergence. The results obtained from this indicate an improvement in design function, with structures created using the new die having a significantly lower “Maximum Damage Value” (stress value which is obtained when visible cracks appear in the structure). From this, the newly optimised die is able to generate extruded designs that do not exceed this maximum damage value, unlike its previous non-optimised solution in which components did not pass this criterion.

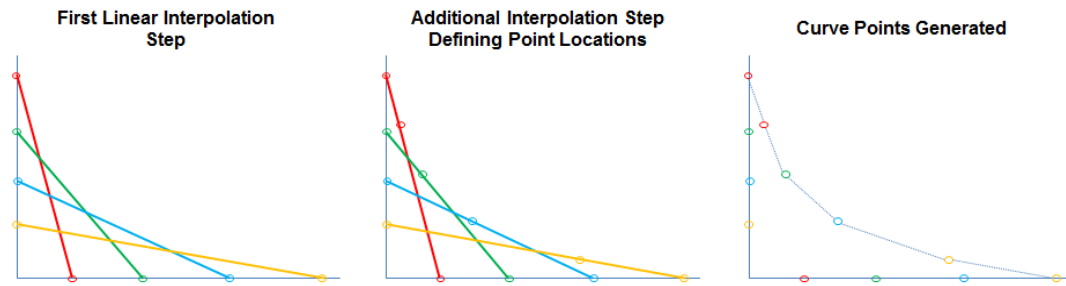


Fig. 9 – Bézier Curve Generation using Linear Interpolation (adapted from Nana et al. (2016))

Several limitations exist with the use of Bézier curves that can limit its usage when compared to the aforementioned NURBS curves. This is predominantly due to how the curves are generated, in which there is a significant lack of freedom to control individual local coordinates in comparison to NURBS curves. The use of linear interpolation means that the modification of one control node will likely influence the surrounding connected nodes and curves, thus changing the overall structure drastically. By inciting a multiplying factor when changing control point positions, the entire curve will be drawn towards this point, thus changing the position of the interpolated curve. This influence on the entire curve can be alleviated by either using higher order curves (see **Equation 2.22** for reference), or by using alternative curve representation methods such as NURBS curves. Increasing the number of control nodes used for the surface can also be considered, but in turn this would make the design more computationally expensive than that of an equivalent NURBS curves model (Deadcodersociety.org 2017).

## 2.3 Overview of Manufacturing Methods in the Automotive Industry

When attempting to bridge the gap between topology optimisation results and the creation of a manufacturable solution, it is important to consider the effects certain manufacturing processes have on component performance and topology. This section will outline several manufacturing methods commonly used in the automotive industry and identify their influence on the development and understanding of generating manufacturable optimisation solutions. It should also be noted that in order to limit the scope of manufacturing processes to review, the focus of research, as mentioned in **Section 1**, is to identify methods relatable to load-bearing components, most of which are sheet metal components located in the main Body in White (BiW) structure.

### 2.3.1 Manufacturing Procedures in the Automotive Industry

Several manufacturing methods exist for the development of automotive components, with several processes being commonly used for a variety of components. As component optimisation usually holds a high focus to the structural performance of a vehicle, the development of the main vehicle structure (commonly manufactured from steel and aluminium materials (Omar, 2013)) will be mentioned within this paper. **Table 1** identifies the most common practices used for the manufacture of automotive components that are part of the structural body. From **Table 1**, it is identified that only two of the identified manufacturing processes relate to the development of components that directly relate to an automotive vehicle's structural performance, namely metal forming and welding/joining. The former holds a

variety of sub-processes that are used to create a large proportion of the structural composition of a vehicle. A focus should initially be taken to the development of optimisation post-processing that can work for individual components rather than full systems (as is the focus with welding methods). It can also be argued that sheet metal manufacturing methods are underrepresented in topology optimisation post-processing considerations (see **Section 3.1**). Due to this, the remainder of this document will prioritise focus towards these sheet metal forming methods. **Table 2** thus presents an overview of common sheet metal procedures used to develop external automotive parts, with considerations of their possible implementation into a post-processor (PP) being identified. **Table 1** also indicates emerging manufacturing methods that could potentially be used for automotive components. Additive Layer Manufacturing (ALM), also widely known as 3D-printing, is an example of one such emerging method, and can be used to generate very complex component geometry which would otherwise be difficult (or even impossible) to produce using standard methods. This process is however very time consuming and not currently suited for large-scale automotive processes. Nonetheless, despite the initial focus of this paper, emerging and transferrable manufacturing methods should not be overlooked.

### 2.3.2 Implementation in Topology Optimisation Techniques

Recent journal papers have proposed alternative topology methods to that of standard mathematical optimisation approaches. These methods, unlike those identified in **Sections 2.1** and **2.2**, focus on improving designs for manufacturability, with specific manufacturing methods being considered within the topology formulation. The following examples outline their processes and identify any outlying limitations:

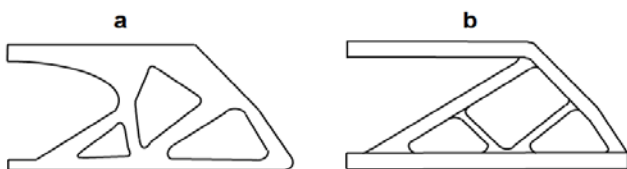
The paper by Zegard (2015), titled, "Bridging topology optimisation and additive manufacturing" identifies a new method for combining SIMP methodology with post-processing procedures to generate feasible additively manufactured structures. It is stated that a greater variety of optimised design structures can be created from additive manufacturing and that it initiates a leap forward in designing suitable structures for medical and automotive sectors. The ability to generate topological solutions from a single structure using one main manufacturing process proves to be more efficient than standard manufacturing methods. Processes other than additive manufacturing may involve the use of multiple manufacturing tools such as lathes, milling machines etc. and would constitute to larger material waste than that of a method that only adds the material when necessary. Additive structures also allow for the user to generate more complex structures that would otherwise be unable to be manufactured if produced through other available processes. **Figure 10** illustrates these differences in the formation of a final manufactured product.

**Table 1 – Overview of General Manufacturing Processes used within the Automotive Industry**

Manufacturing Process	General Process	Potential Applications	Comments
Metal Injection Moulding (Casting)	Uses powder metal heated and compressed into a mould to create solid components (GKN 2013).	Interior vehicle components including the engine block, wheels and carburettor (The Metal Casting 2017).	Automotive components created using this method are typically not structurally-related and do not consist of any external body panels.
Sheet Metal Forming	Bending/shaping of sheet metal to form components (Groover, 2011).	Typically used for external automotive body panels and structural body.	Used for creating components involved with automotive structural performance (see <b>Table 2</b> ).
Welding/Joining	The coalescing of two metal components by use of heat, pressure, and in some instances a filler material (Groover 2011).	Used to weld sheet metal components together, such as in external automotive body panels (Devarasiddappa 2014).	Used to fuse already manufactured sheet metal components. Higher importance should be made to the generation of individual components, especially in an optimisation sense.
Machining	Use of high powered tools such as drills/lathes to cut and shape metal into its desired structure. Designs are generally drawn using computer software (ThomasNet 2017).	Creation of precisely cut components such as valves, pistons, headlight housings and aluminium wheels (Cam-Machine 2015).	Typically not used for structural components or other space frame/chassis components.
Metal Additive Manufacturing (3D Printing)	Uses a specially designed printing machine that forms layers of heated powdered metal to create a 3D component (Metal-AM 2017).	Generation of cylinder heads, intake manifolds, and air vents (Deloitte University Press 2014).	Process may be suitable for developing components that would otherwise be casted but shows little incentive for creating sheet metal components due to practicality when compared to traditional methods.

**Table 2 – Overview of Sheet Metal Manufacturing Processes for External Automotive Components**

Process	Process Overview	Practicality of Method	Consideration in PP
<i>Sheet Metal Stamping</i>	Collective term which outlines the process of deforming of cold sheet metal using a die. Three variations of this process include cutting ( <i>Shearing, Blanking and Punching</i> ), folding ( <i>Bending</i> ) and stretching.	Most common process of automotive part generation. Finish quality is generally smooth but can invoke thickness changes where deformations (bends etc.) are present.	Suitable for sheet components with thicknesses ranging from 0.1-6.5mm → consideration of thickness parameters may be needed to identify these tolerances (Omar 2013).
<i>Shearing, Blanking and Punching</i>	<b>Shearing</b> - cutting of sheet metal components using power (squaring) shears  <b>Blanking and Punching</b> – Cuts a shape by removing material from the sheet metal (Groover 2011)	Used in large-scale vehicle manufacturing. Mostly concerns feature fitting (e.g. hole creation) to which casting methods will be used for the initial shape generation.	Tolerancing is required for all three methods to improve geometrical accuracy as material shearing creates small fractures which reshapes sections of components.
<i>Bending</i>	Working sheet metal to create a curve along a neutral axis. Generally performed with punches which apply a force to a sheet resting on a die (Groover 2011)	Very common process in automotive manufacturing Useful method to create bends in a component without the need for casting.	The sheet metal used will stretch and compress about the bending axis, leading to structural performance changes.
<i>Drawing</i>	Creation of concave/enclosed parts by pressing sheet metal into a die cavity. Used for forming cup/box shaped parts and is achieved through using a concave part (die) to rest the material, a blankholder to secure the sheet metal and a punch to press and extrude the material (Groover 2011).	Commonly used in automotive industry. This process will cause the pressed object's thickness to decrease after it is pressed (Groover 2011), resulting in structural performance changes	Thickness tolerances should be adhered to due to the stretching and pressing. Thickness changes may also result in changes to structural performance.



**Fig. 10** – a) Additive Layer Manufacturing (ALM) truss component formed as one solid piece vs b) Truss component manufactured from individual machined parts welded together

Several limitations are still present in this methodology despite the gap between topology optimisation and manufacturable components being reduced by this technique. The first issue to consider is that a manual post-processing step is still needed to correct geometry before it can be manufactured. Post-processing steps would involve the user manually changing the result geometry of the FE optimised solution such that it can be manufacturable. Issues such as disconnected elements

will also need to be addressed. Additionally, the efficiency of additive manufacturing is still poor when compared to other large-scale industrial methods. Until this bottleneck is removed, the feasibility to use this process for large-scale development, especially in automotive sectors, is very limited.

Liu and Ma (2015) discuss the inclusion of machine feature cutting for level set topology optimisation in the paper titled, “3D level-set topology optimisation: a machining feature-based approach”. This paper proposes the use of heuristic methods in the form of a level set optimisation process, whilst also running a shape optimisation to refine the design. The initial process involves the generation of an FE optimisation solution using topology level set optimisation. This utilises a SIMP-based approach with level set refinement processes performed at the end of every iteration. The process uses what is known as a Hamilton-Jacobi formulation, which allows for the FE geometry to split and



combine over the iteration process, thus updating the level set function (Sigmund and Maute 2013). In order to include the level set function into the optimisation procedure, two functions, known as the Heaviside function (2.26) and Dirac delta function (2.27), are defined as follows:

$$\begin{aligned} H(\phi) &= 1, & \phi &\geq 0 \\ H(\phi) &= 0, & \phi &< 0 \end{aligned} \quad (2.26)$$

$$\delta(\phi) = \frac{\delta H(\phi)}{\delta \phi} \quad (2.27)$$

Equations 2.26 and 2.27 can then be interpreted in relation to the open front, or the area outside of the physical component space,  $\Omega$ :

$$\Omega = \{X | H(\phi(X)) = 1\} \quad (2.28)$$

$$\delta\Omega = \{X | (\delta(X)) > 0\} \quad (2.29)$$

Equations 2.28 and 2.29 can then be formulated into a Hamilton-Jacobi format such that it can determine the propagating speed of the boundary, i.e. the rate at which the boundary updates per iteration. The Hamilton-Jacobi format is shown in Equation 2.30:

$$\frac{\delta\phi(X)}{\delta t} = V_n |\nabla\phi(X)| \quad (2.30)$$

$V_n$  references the boundary speed in the normal direction to the geometry, to which  $n$  is defined as follows:

$$n = -\frac{\nabla\phi(X)}{|\nabla\phi(X)|} \quad (2.31)$$

This boundary speed is calculated within the optimisation formulation and is updated for every iteration, thus updating the boundary (Liu and Ma 2015).

This process further includes the incorporation of a “feature-fitting library”, which identifies specific geometrical features in the topological design and replaces them with a set of pre-existing CAD designs with machining feature finishes. Whereas this method closes a significant gap between optimised designs and manufacturability, it is significantly limited by the small number of feature fitting methods included within the library. This would mean that unless the library is significantly expanded, the types of solutions will be limited to a small variation of machining features. As metal forming techniques are more commonly used for external vehicle components, it may be more suitable to favour these over machining methods. Furthermore, the optimisation process considers the use of an integrated SIMP solver with a level set heuristic methodology. It may be suitable to separate the mathematical topological processes from the refinement steps such that they can be tailored towards a variety of manufacturing methodologies. This understanding is discussed and justified further in Section 4.

### 2.3.3 Representation of Manufacturing Constraints

In order to generate manufacturing-ready models from optimisation files, it is important to outline suitable manufacturing constraints that can be used to refine a topological structure. Identification of measurements and tolerances that are used in existing processes will provide a better understanding into the limitations of model predication and in turn aid in the development of future post-processing algorithms. As outlined in Section 2.3.2, a significant level of focus for this review will be on automotive manufacturing methods, to which the consideration of metal forming methods will take initial priority. As identified in Section 1, the reasoning for the emphasis on sheet metal formed components is due to this paper's focus on refining optimised automotive

components used in crash and load-bearing functions. From this, the following information identifies potential parameters that could be included within either a mathematical or heuristic post-processor:

### Material Specifications and Machine Tolerances

The first set of constraints will concern sheet metal component manufacturing properties and tolerances. As mentioned in Section 2.3.1, metal forming methods for automotive sheet metal components are the most prominent and favoured manufacturing process used. Most external automotive panels are made from steel and aluminium due to their low cost and ease to form and weld into larger structures (whilst maintaining high structural stiffness). Commonly used steel grades include J2329 grades 1 to 5, which can be cold or hot-rolled. Popular aluminium grades typically come from the 2000, 5000, 6000 and 7000 series, with the 2000 series producing a higher stiffness, thus being more commonly used than the other grades (Omar 2013).

When considering the metal forming techniques mentioned in Section 2.3.1, certain dimensional restrictions will be involved due to the limitations presented in standard metal forming dies. Many of the necessary parameters are quantifiable and can potentially be easily implemented into a post-processor that can refine a model for these manufacturing considerations. Table 3 outlines a selection of tool and die limitations for aluminium components when punching and drawing sheet metal (Omar 2013):

**Table 3 – Die and Punch Specifications for Aluminium Components (adapted from Omar (2013))**

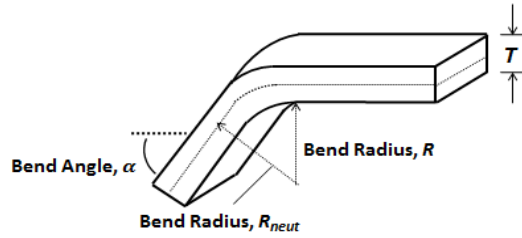
Drawing design method	Quantified Value
Punch Radius	$4 \times t_{panel} - 12 \times t_{panel}$
Die Radius	$4 \times t_{panel} - 12 \times t_{panel}$
Draw Bead Depth	$6 \times t_{panel} - 8 \times t_{panel}$
Draw Bead Radius	$5 \times t_{panel} - 7 \times t_{panel}$
First Draw	$1.1 \times t_{panel}$
Second Draw	$1.1 \times t_{panel} - 1.5 \times t_{panel}$
Third or Subsequent Draw	$1 \times t_{panel} - 1.2 \times t_{panel}$

$t_{panel}$  defines the thickness of the aluminium sheet before the drawing or punching methods are performed. For the case of multiple drawing methods (second/third draw) the thickness will be that of the material before this additional draw is performed. It is assumed that the ratios defined in Table 3 are sufficient requirements to prevent issues such as large thickness changes or any unwanted material distributions (i.e. no structural performance changes that are not directly related to the methods mentioned). More information related to these defined thickness ratios are described into further detail in the European Aluminium Automotive Manual (European-aluminium.eu 2017).

Considerations to bending angles can include the maximum allowable bending angle in the sheet material. This can be extracted (and implemented into suitable software) by calculating the allowable strain using the formula shown in Equation 2.32:

$$\varepsilon = \frac{R - R_{neut}}{R_{neut}} \quad (2.32)$$

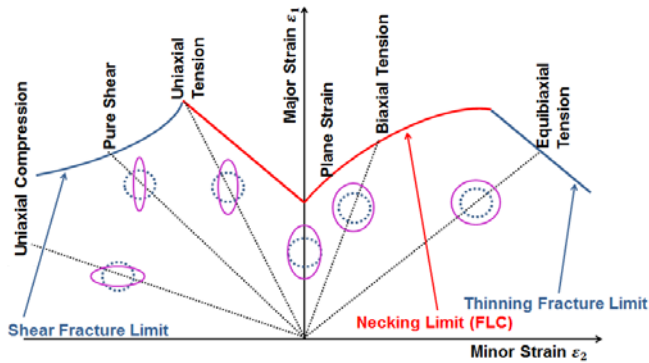
Where  $R$  is the radius of the bend, and  $R_{neut}$  is the radius on the neutral axis (see Figure 11). Further details can also highlight suitable machinable thicknesses for sheet metal components, with Omar (2013) stating that workable thicknesses should range between 0.1-6.5mm.



**Fig. 11** – Locations of bending radius,  $R$ , and indication of material thickness,  $T$ , for a formed metal component (adapted from Kalpakjian and Schmid (2008))

### Forming Limit Diagram (FLD) and Material Property Considerations

A Forming Limit Diagram is a tool used to predict the material behaviour and failure criteria for the forming of sheet metals and provides a graphical interpretation of the stresses and strains that act on the component. The FLD allows for the identification of whether a component will fail or remain safe by relating to strain data induced on the component due to stretching. A typical setup to record sheet metal strains due to deformation include marking a metal sheet with circles, identifying locations for which strain measurements are to be taken. The material is then stretched across a die and is subjected to a specified load or bend shape. When the metal is stretched, the circles are deformed into an elliptical shape. The change in length in the major axis (direction of stretching) and minor axis (90 degrees to the direction of stretching) is then recorded and graphically represented in a Keeler-Goodwin diagram (nptel.ac.in 2017). The basic design of this diagram is illustrated in **Figure 12**, in which major and minor strains are plotted. This highlights a relationship between major and minor strains, by which a “line” can be drawn to determine a typical “safe zone” for deformation and predict its failure:



**Fig. 12** – Keeler-Goodwin Forming Limit Diagram for a sheet metal component (adapted from Researchgate.net 2017)

The “safe zone” for deformations lies below the shear/necking/fracture limit line. Visual representations of the changes made to the marked circle on the sheet are also displayed, highlighting the physical material changes in necking and stretching terms. Several aspects are used to influence the position of the safe zone along the diagram, with material composition, thickness variations and strain hardening values all influencing the feasibility of a structurally stable component (nptel.ac.in 2017).

The use of FLD should ideally be considered when refining topology optimised models in order to ensure the feasibility of a given structure. This process could be considered within a topology solver itself, but can also be utilised within the refinement steps to correct any topology in a more localised manner. Currently, this process has not been implemented into topology solvers or post-processing steps but may be seen as a necessary inclusion to improve model accuracy.

### Bending Angles

The overall estimation of the allowable sheet metal radii before material thinning occurs can be formulated, and identified in **Equation 2.33** (european-aluminium.eu 2017):

$$R_{\text{at critical thinning}} = \frac{h}{2n} \quad (2.33)$$

In which  $h$  represents the height of the bend section above its original position and  $n$  relates to the material's hardness coefficient, for which the following coefficients apply within automotive components:

**Table 4** – Hardness coefficients for common materials (adapted from Callister and William (2005))

Material	Hardness Coefficient, $n$
Low-Carbon Steel	0.21
4340 Steel Alloy	0.12
304 Stainless Steel	0.43
2024 Aluminium Alloy	0.17

### Springback

Material springback is another process not seen in commercial optimisation solvers or post-processors and occurs after the bending process, in which the initially created bend will “pull” itself towards its original undeformed shape, thus reducing the degree of the initial angle. Implementation of the springback factor,  $K_s$  is shown in **Equation 2.34**:

$$K_S = \frac{\alpha_f}{\alpha_i} = \frac{\left(\frac{2R_i}{t}\right) + 1}{\left(\frac{2R_f}{t}\right) + 1} \quad (2.34)$$

Where  $\alpha_f$  and  $\alpha_i$  represent the current springback angle and the original angle, respectively,  $R_f$  and  $R_i$  represent the current and original radius of the bend and  $t$  is the unchanged thickness of the metal sheet. When  $K_S = 1$ , it is considered that there is no springback acting on the material. It is also known that springback is less severe in aluminium and austenitic stainless steel, with greater springback in steels that have a much greater hardness value (Kalpakjian and Schmid 2008). It would therefore be an important consideration to record material information when measuring this effect. It is shown that the main consideration for the determination of both springback and bending angles relate to specific thickness and radii parameters of the material before deformation. As this process is also absent from current commercial software, future implementation of this process should be considered.

### Possible Application in Mathematical and Heuristic Methods

The previously mentioned constraints can be included within combinations of existing mathematical solvers and emerging heuristic solvers. Mathematical attempts to constrain geometry can be found within existing solvers that utilise SIMP methodology. Several existing software tools can, for example, identify and set thickness constraints for components, identifying specific characteristics such as beam and wall thicknesses. The ability to transfer this definition of thickness criteria to a post-processor can be seen as very beneficial for ensuring thicknesses are attained throughout the clean-up process. Additional topological features that can potentially be implemented to close the gap in optimisation and manufacturability can be the use of “Minimum Member Size”, MINDIM, and “Maximum Member Size”, MAXDIM, functions, which are used within programs for certain mesh pre-processors. MINDIM is a function that controls the smallest dimension of a



topological structure and aims to reduce the level of checkerboarding present in a VDM topological solution. This is achieved by setting suitable parameters that penalise the generation of small members of a given threshold value and also reduces the number of intermediate elements, creating a more “binary” solution. MAXDIM, as its namesake, controls the maximum dimension size of members within a topological structure. This desires to create a structure that has much smaller detailed members, to which it is required that the MAXDIM value is at least six times the length of the average element size. This method thus performs better when using a much finer mesh. This consideration can potentially be transferred into a proposed post-processor to control the number of holes generated within a topological structure (Altair University 2015).

Several heuristic methods such as the level-set method (**Section 2.2.1**) can also be considered when defining manufacturing considerations. Identification of discernible borders and feature shapes can ensure that the definition of bend angles and holes becomes more regimented. This can then make the process of defining maximum bend angles, for instance, much easier.

Furthermore, certain existing software extensions present in some pre-processors can be integrated into a proposed solution methodology to consider certain manufacturing features. For instance, specific optimisation solvers can account for design features such as extrusions, stampings, mouldings, as well as other commonly used manufacturing methods. These processes are however not all fully automated, with some functions requiring further manual corrections (Altairhyperworks.com 2017). These tools are also designed to consider only one type of manufacturing feature at a time, nonetheless it could be suggested that some of these processes can be integrated within a combined solver in the future.

## 2.4. Refinement Process Review

The previously described optimisation and refinement processes from **Section 2.2** outline several current and emerging methods seen in optimisation solvers. These prominent refinement methods can be classified as processes that are currently used within existing commercial software and those that utilise new algorithms personalised towards solving particular manufacturing problems. **Table 5** presents an overview of these contrasting methods and their feasibility in generating a suitable refined optimisation model, whilst also considering available compatibility with sheet metal forming and thickness detections. It can be deduced that even though refinement steps have been vastly improved with the creation of these new methods, they do not consider specific manufacturing features for sheet metals (the exception being the inclusion of certain add-ons for specific commercial software for the pre-processing of SIMP models).

Other existing and emerging methodologies for topology optimisation refinement should be considered for the future development and improvement of generating manufacturable designs. Recent methods, as indicated in **Section 2.2**, include Isogeometric analysis, which considers using CAD geometry in the initial analysis stage as opposed to a traditional mesh, thus creating a smoother optimised result. More recently, focus has been drawn towards the development of unique optimisation algorithms suited to solve specific tasks, such as those

relating to manufacturing considerations, as well as considering other areas such as multi-objective optimisation (USACM 2017). Automation of optimisation processes is also a vastly emerging consideration, with many unique solvers being developed that combine traditional optimisation methods with refinement processes for certain situations. Examples include works by Yi and Kim (2016) for defining holes using shape optimisation, Zegard and Paulino (2015) who integrate optimisation with additive manufacturing and Liu & Ma (2015) who remove several post-processing steps by replacing results with pre-existing designs (see **Table 6, Section 3.1**, for full list). An appreciation of these solvers is shown in **Table 5**. In accordance to this understanding, it can be determined that despite the variety of emerging topology methods, significant manual input is still needed to generate a manufacturable solution. Emerging methodologies are thus aiming to address this issue.

## Consideration of Practical Automotive Problems

Defining the most desirable optimisation methodology can be highly important when optimising sheet metal components. Structurally important components, such as those used to construct the body in white (BiW) of a vehicle, are mostly made from sheet metals. Examples include the construction of an automotive B-pillar or even a side impact beam, with aluminium being the choice material for most commercial vehicles (see **Section 2.3.1**). During manufacturing, these components are exposed to stress and strain, e.g. from a stamping process as mentioned in **Section 2.3.3**. The component may contain holes or other cut sections, for which a detailed set of optimisation constraints are needed. Holes and cut features will need to account for bolts and fixture placements, ensuring that the refined design can still interact with the surrounding components as originally intended. These considerations can be made by implementing the processes mentioned in **Section 2**, as well as identifying specific geometrical modification processes that can aim to generate smoother holes and bend angles. Identification of currently available methods are shown in detail, within **Section 3**.

## 3 Recent Implementations of Topology Optimisation & Manufacturability

Recent methods have shown an increase in interest for optimisation refinement and integrated post-processing and topology optimisation solvers. This section highlights specific literature and methodologies used in relation to automation in post-processing and the refinement of topology optimisation results. Key papers will be identified to indicate an understanding as to how some of the methods identified in **Section 2** have been adapted to real-world applications:

### 3.1 Summary of Relevant and Recent Papers

Several journal papers address issues regarding the development of a manufacturing-focussed automated post-processor. The following identified papers, shown in **Table 6**, were scrutinised based on relevance to key considerations. These include the level of automation involved in the proposed methods, how they incorporate manufacturing considerations, the types of files that are able to be refined and whether a separate dedicated post-processing step is used after the main optimisation.

**Table 5 – Critical Review of Existing Topology Refinement Processes**

Optimisation Refinement Method	Description	Refinement Strategy	Pros	Cons	Possible use with Forming Methods?	Considers Material Thickness?
VDM SIMP	Redistributes material densities in relation to the loads applied on the structure, forming a model with the lowest possible structural density.	Calculates the material stiffness matrix for each element in relation to an applied load and constraints if applicable. This solution runs through an iterative process and determines suitable material densities based on internal material stresses.	<ul style="list-style-type: none"> <li>Used in most commercial optimisation software</li> <li>Variable density solution is tailored to the user's own preferences.</li> </ul>	<ul style="list-style-type: none"> <li>As it generates a solution of variable density, significant manual input is needed by the user to make any result representative of a manufacturable solution.</li> </ul>	No	Yes
BESO	Removes or adds individual elements of a structure in order to identify its optimum structural shape.	Following a similar iterative process to VDM SIMP, elements are evaluated depending on their structural importance. Instead of creating different densities, elements are removed or added based on their structural importance.	<ul style="list-style-type: none"> <li>Ability to generate a binary solution model that represents a more reputable solution than that of VDM solutions.</li> </ul>	<ul style="list-style-type: none"> <li>Evolutionary algorithms involve additional initialisation steps, including running the optimisation algorithm for each iteration. This in turn involves greater computational cost.</li> </ul>	No	Yes
Level Set Topology Optimisation	Defines a suitable solution by removing or adding elements and ensuring discernible borders are established around the edges of the design.	Follows an iteration loop that represents the initial process of the topology solver used in standard VDM SIMP methods. An additional refinement step is also included in these iterations.	<ul style="list-style-type: none"> <li>Creates more refined binary solution than that of a VDM solution</li> <li>Better consideration of component border definition</li> </ul>	<ul style="list-style-type: none"> <li>Whereas solutions are more representative than other existing methods, this process still does not generate immediately "manufacturing-ready" solutions.</li> </ul>	No	Yes

Consideration into the ability to transfer these methods to sheet metal refinement is also identified, with suitable weightings made to identify the most relevant studies.

**Table 6** clearly indicates that there is currently a small focus towards refining topology optimised designs for specific manufacturing processes, with only four recent papers tailoring their development towards these considerations (papers 3 (Liu and Ma 2015), 5 (Chacón et al. 2014), 7 (Zegard 2015) and 11 (Lee et al. 2012) in **Table 6**). Furthermore, current research indicates that there is no substantial development for the consideration of metal forming methods in topology refinement. An issue also indicated from **Table 6** is that there are few examples where the post-processor is separate from the topology solver, in which papers 1 (Yi and Kim 2016), 5 (Chacon et al. 2014), 6 (Lin and Chao 2000), 7 (Zegard and Paulino 2015) and 8 (Mandhyan et al. 2016) in particular focus on integrating the refinement methods within the topology optimisation solver (as opposed to separating these steps). It is important to ensure that the post-processor is separate from the topology solver such that multiple types of topology solutions (Binary and VDM – **Section 2.1**) can be refined. As most examples only either consider refining binary or greyscale models, and never both, this can be seen as a key important area for development. Furthermore, paper 4 (Koguchi and Kikuchi 2006) and paper 9 (Nana et al. 2016) identify unique algorithms which integrate mesh smoothing methods with existing topology optimisation processes. This can be seen as an important step in developing an automated processor to refine structures, but they both sadly fall short of including specific manufacturing processes within their refinement steps. Paper 10 (Lee et al. 2016) also dismisses the use of any specific manufacturing considerations, but does, as with papers 4 and 9, identify a unique refinement step to generate a smoothed geometrical solution.

The three highlighted papers in **Sections 3.2-3.4** (numbered as papers 1 (Yi and Kim 2016), 2 (Kang and Youn 2016) and 3 (Liu and Ma 2015) in **Table 6**) indicate the strongest cases for development of an automated post-processor, tailored towards manipulating an optimisation solution into a usable manufacturable design. Unlike other identified papers in **Table 6**, papers 1, 2 and 3 detail post-processing methods that consider

specific manufacturing processes, such as for beams (paper 1), hole generation in 2D sheets (paper 2) or 3D machining tool features (paper 3). Understanding how manufacturing features are considered within topology solvers is highly important as it will provide an understanding of the existing methodologies and identify any potentially transferrable relation to sheet metal forming. Detailed outlines of these papers, bringing attention to automation and potential room for improvement, are discussed in the following sections.

### **3.2 “Identifying boundaries of topology optimization results using basic parametric features” (Yi and Kim 2016)**

The first identified paper concerns the use of an automated post-processor that can bridge a “serious gap” between greyscale topology optimisation results and manufacturable CAD designs. The paper proposes a flowchart to outline the automated post-processing program, which consists of a “geometric features identification” of the greyscale (VDM) topology model, followed by a parameterisation of the elements to create a discernible structure with defined borders. This is then followed by a shape optimisation step to clearly define the new geometry. **Figure 13** highlights the overall flowchart process of this post-processor.

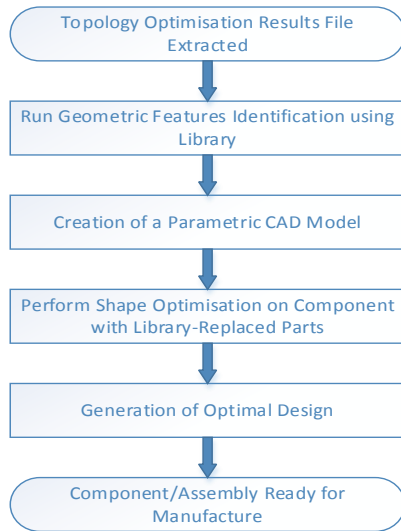
The post-processor indicates a method of solving an issue found when using VDM and SIMP-based topological solvers. This involves attempting to create a suitable manufacturable shape whilst accounting for the variable material densities in the solution mesh. The use of active contours (also known as snakes), defined as splines that identify edges, are utilised within this algorithm. The snakes are used to determine edge boundaries and holes for a 2D topology mesh, in which a computational code is created to define the snakes that fully enclose a hole or edge. By identifying clear, smooth boundaries within the optimised structure, issues previously present in existing optimisation solvers, such as the checkerboard effect and variable density solutions (**Section 2.1**) will be alleviated.

**Table 6** – Overview of Papers Relating to the Development of Automated Post-Processing and Refinement of Models

		Automated Process	Automotive Manufacturing Considerations	Considers Metal Forming Methods	Potential translation to Metal Forming Processes	Multiple File Input	FEA File Output	CAD File Output	Separate Post-Processor	Includes Component Refinement	
No.	Journal Paper	6	7	9	8	4	2	1	5	3	Total
1	Identifying boundaries in topology optimisation results using basic parametric features (Yi & Kim 2016)	✓	✓	–	✓	–	–	✓	✓	✓	30
2	Isogeometric topology optimisation of shell structures using trimmed NURBS surfaces (Kang & Youn 2016)	✓	–	–	✓	–	–	✓	–	✓	18
3	3D level-set topology optimization: a machining feature-based approach (Liu & Ma 2015)	✓	✓	–	✓	–	✓	✓	–	✓	27
4	A surface reconstruction algorithm for topology optimisation (Koguchi & Kikuchi 2006)	✓	–	–	–	–	–	✓	–	✓	10
5	Integration of topology optimized designs into CAD/CAM via an IGES translator (Chacon et al. 2014)	✓	–	–	–	–	–	✓	✓	✓	15
6	Automated image interpretation for integrated topology and shape optimization (Lin & Chao 2000)	✓	–	–	–	–	✓	–	✓	✓	16
7	Bridging topology optimisation and additive manufacturing (Zegard & Paulino 2015)	–	✓	–	–	–	–	✓	✓	✓	16
8	A novel method for prediction of truss geometry from topology optimisation (Mandhyan et al. 2016)	✓	–	–	–	–	–	✓	✓	✓	15
9	Towards adaptive topology optimisation (Nana et al. 2016)	✓	–	–	–	–	✓	–	–	–	8
10	Isogeometric topological shape optimisation using dual evolution with boundary integral equation and level sets (Lee et al. 2017)	✓	–	–	–	–	–	✓	–	✓	10
11	Die shape design of tube drawing process using FE analysis and optimisation method (Lee et al. 2012)	✓	✓	–	–	–	–	✓	–	✓	17

**Key:**

✓ Included  
– Not-Included

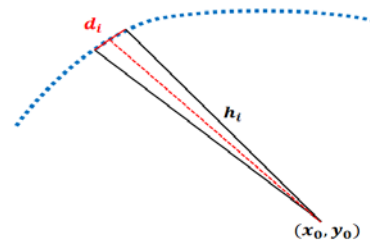


**Fig. 13** – Flowchart overview of process for combined optimisation solver and post-processor. (adapted from Yi and Kim (2016))

The identification of various geometrical features involves the incorporation of multiple formulae within the generated code. These formulae relate to specific geometrical features for 2D structures, including hole detection, identifying straight lines and determining closed outer boundaries. **Equation 3.1** highlights one of these proposed formulae, which outlines the location of a snake for a circle about an origin  $(x_0, y_0)$ :

$$(x_0, y_0) = \left( \frac{1}{n} \sum_{i=1}^n x_i, \frac{1}{n} \sum_{i=1}^n y_i \right) \quad (3.1)$$

In which  $n$  refers to the number of snake points generated in the closed boundary, and  $x_i$  and  $y_i$  refer to the coordinates on the  $i^{th}$  snake point. After identifying the origin node location, the distance from this point to two adjacent snake points can then be triangulated and subsequently measured. To clarify, **Figure 14** shows the information currently available to the user and the positions to be identified,  $d_i$  and  $h_i$ .



**Fig. 14** – Set of defined boundary points (adapted from Yi and Kim (2016))

These points are then triangulated with the origin node to calculate the distances  $d_i$  and  $h_i$ :

$$d_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2} \quad (3.2)$$

$$h_i = \frac{|k_i x_0 - y_0 + y_i - k_i x_i|}{\sqrt{1 + k_i^2}} \quad (3.3)$$

where  $k_i = (y_{i+1} - y_i)/(x_{i+1} - x_i)$ . After identifying the triangulated distances, the perimeter,  $P$ , and area,  $A$ , of the triangle can be deduced as follows:

$$P = \sum_{i=1}^{n-1} d_i \quad (3.4)$$

$$A = \sum_{i=1}^{n-1} \frac{1}{2} d_i h_i \quad (3.5)$$

Using the perimeter and area it can then be determined whether these two points lay on a circle section or not. This is defined by calculating the roundness of the loop that lies around the origin  $(x_0, y_0)$ , and is formulated as follows:

$$m = \frac{4\pi A}{P^2} \quad (3.6)$$

If the calculated value for  $m$  is close to 1, it is assumed that the enclosed section of points about the origin node are in the shape of a circle. Similar methodologies also exist for the determination of straight lines and enclosed, non-round surfaces, which will not be further mentioned in this literature. By identifying finite closed lines from the mesh, a geometry clean-up is performed by either replacing some uneven edges with straight lines or by identifying the locations of beams within the structure and replacing the original geometry with these.

The work of Yi and Kim indicates arguably the farthest development for the creation of an automated post-processor, involving no significant user input aside from referencing the input optimisation file. It should be noted that there are, however, several aspects of this automated post-processor that can be improved upon for a more convenient transition to a manufacturable component. Firstly, there should be consideration that the solution generated only consists of a line model, with the 2D component solution not consisting of any CAD surfaces. It is therefore expected that the user generates any 2D CAD surface manually based on the lines generated by the post-processor. The algorithm itself, where useful in improving the quality of greyscale elements, is solely limited to improving only this type of optimisation model. For instance, it is not known whether this method will work with optimised mesh files generated using evolutionary optimisation, which creates a binary material density solution. Additionally, despite this refinement process aiding in the creation of more manufacturing-friendly designs, it does not consider specific manufacturing methods, nor does it consider the changing stress distributions that occur on the component over the optimisation process (see **Section 2.3.3**). The ability to accurately record stress values is an important consideration to make as it will provide the user with an understanding as to how performance is altered over the optimisation process and identifying if the product would underperform in its required function.

### 3.3 “Isogeometric topology optimisation of shell structures using trimmed NURBS surfaces” (Kang and Youn 2016)

This paper presents a novel topology optimisation solver that combines the level set method for image recognition with shape optimisation techniques. Shape optimisation uses a different technique to topology optimisation, in which the shape of the structure is changed (but mass and other topological properties are not necessarily changed) in accordance to certain loading parameters (Christensen and Bastien 2015). The optimisation and refinement processes are both carried out within the solver, with no post-processing steps considered. In a similar manner to that indicated in **Section 2.2.2**, NURBS surfaces are used for determining the edges of a greyscale 2D topology optimised mesh model. In this case, as well as generating refined CAD lines for the new

geometry, a NURBS surface has been created. This involves “projecting” the refinement lines and using them as cut lines for a new 2D surface. Formulation of the IGA process is highlighted in **Section 2.2.2**. This surface will then be cut according to the projected lines generated from the NURBS curves, thus creating the new surface. After the meshless surface is generated, further analysis and refinement is performed using Isogeometric Analysis.

Isogeometric Analysis (IGA) is an emerging method which utilises the CAD geometry (instead of an FE mesh) for the recording of structural analysis data, such as von Mises stress. TSA optimises the CAD surface by “cutting” the NURBS surface (see **Section 2.2.2**). This follows a different methodology than that of standard topology optimisation methods, which considers the material density as the main variable. When optimisation iterations are performed, NURBS curves projected onto the cut surface will be continuously updated, re-adjusting the topology of the surface. Conversely, additional holes may be added to the surface over several iteration processes to increase the number of openings on the surface wherever necessary. The proposed methodology then follows with a shape optimisation to update the stress distributions acting on the component. As with paper 1 mentioned in **Section 3.2**, the use of this alternative method will alleviate certain geometrical issues that would occur in existing optimisation algorithms, such as element checkerboarding and variable material densities.

Whereas the methodology presented by Kang and Youn indicate a novel idea, several limitations arise when applying this process to component manufacturability. It should be noted that no particular manufacturing examples were used within the case studies in this paper, indicating that the methodology has not yet been extended to consider specific metal forming processes, especially those common to the automotive industry. Also, it should be mentioned that as the proposed algorithm does not perform an analysis or optimisation on a finite element mesh, it can be assumed that several discrepancies may exist between the initial input and final CAD model. It should therefore be recognised that the structural analysis results obtained directly from the CAD geometry are most likely not as accurate as those obtained via FEA. The generation of a density-based mesh is currently a widely used methodology, especially within the automotive industry, and will be needed to determine reliable and replicable results. From this it is not clear whether any discrepancies are present or if using different optimisation files (Binary or VDM) will affect the refinement process. It is desired that any automated post-processor should be able to generate results for a variety of optimisation file types. Creation of only a CAD solution can limit the usability of the results and will prove difficult to evaluate its validity if not further manually tested.

### 3.4 “3D level-set topology optimization: a machining feature-based approach” (Liu and Ma 2015)

This paper by Liu and Ma considers the use of level-set topology optimisation methods for the development of 3D components created using 2.5D (2D models with an extruded height) machining methods. The proposed algorithm constitutes of running, in parallel, a feature fitting level-set topology optimisation and shape optimisation. Use of the level-set formulation allows for the boundaries to combine and split whenever the topology needs to be updated, thus providing flexibility when combined with topology optimisation methodology. When defining the desired geometrical features to be used in an optimised solution, a “Polyline-arc Profiling

machining feature library" was used. This feature library contains a series of pre-set component section designs, which will be used in the final optimised model wherever there is a match to the original topology. To be able to include these additional features, a modified version of the level-set topology optimisation algorithm is used. Several angles and shapes are to be recorded after a structural analysis is performed, and will be replaced with the pre-existing designs from the feature library. Unlike the previous journal examples, the proposed algorithm reads a 3D finite element model (2D features with an extruded surface) as the input and produces an optimised 3D mesh design as the output file. This paper also includes methods not seen in other papers in that it considers model preparation and optimisation specifically for machining features. It is also necessary for the user to define the desired draw angle for the 2.5D shape, as the machine cutting feature can only be applied in certain directions.

The proposed model, where suitable for certain machining methods, may not necessarily be for other manufacturing methods, especially those used in the automotive industry. As the solver includes several pre-set feature designs for the optimised component, it may be considered that the solutions generated can be very limited, due to the fact that they are tailored towards producing a solution based on a specific manufacturing method. Another factor contributing to the solver's lack of customisability can be that the refinement stages are integrated into the optimisation steps, leaving no dedicated involvement of a post-processor. It is stated in **Sections 2.1** and **2.2** that a separate post-processor will improve upon the tool's customisability of results and allow for a greater variety of input files to be considered. However, despite these criticisms, the implementation of a database of pre-existing designs should not be ruled out, as this feature may help with the manufacturing of standardised designs commonly seen in mass-produced components. This can be highly relevant towards improving production costs for products, especially those in the automotive industry.

## 4 Conclusions & Further Research

Several methodologies exist for optimising the topology of components, including the Variable Density Method, which re-distributes element densities, and evolutionary optimisation which removes or adds elements from the structure using the full material density. Recently emerging heuristic (iterative) processes such as Level Set topology optimisation (Challis 2009) have been used to identify discernible borders within a structure, refining FEA meshes during the optimisation process.

These optimisation techniques have proven their relevance in defining robust structures but still hold their own imperfections, with their ability to generate CAD/FEA solutions that can be directly manufactured being questionable. Mathematic and heuristic solutions both require a significant level of interpretation and manual post-processing to translate topology results into manufacturable representations of components, which consequently is a significantly long and time-expensive process. Methods such as level set optimisation may help reduce post-processing time by creating a more refined model, but as with other existing methods consideration to specific manufacturing processes is absent. Recent works highlight the need and implementation of automated post-processors but either fall short of generating full CAD or FEA solution files or they do not consider implementing multiple automotive-specific manufacturing processes. Recent algorithms attempt to address specific processes within its post-processing,

with focusses specifically limited to machining features and beam representation. Because of this, there is, at time of writing, no significant consideration for representing sheet metal formed products within any post-processing steps. This can be seen as a significant shortfall due to the widespread use of sheet metal manufacturing, for example, within automotive sectors. Further development to optimisation processes that can improve representation of different manufacturing methods could be to separate the main optimisation solver from the refinement post-processor. This would allow the post-processor to identify multiple types of topology solutions (binary and variable density models) instead of limiting its use to just one methodology, and allow for both of these solution types to be refined for specific manufacturing considerations. Inclusion of multiple optimisation solution types within one post-processor is also not currently supported by existing literature. If this were to be considered in the future, it may encourage FEA users involved in large-scale product development to utilise software-driven processes, instead of the manual methods currently used in most modern automotive industries.

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